DETERMINING THE THERMAL RESISTANCE OF LOW-TEMPERATURE HEAT PIPES

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A method is proposed for calculating the thermal resistance of low-temperature heat pipes. The results are compared with test data.

The heat transfer through heat pipes is affected by several parameters, including the thermal resistance of the pipe walls and of the porous wick saturated with liquid, the temperature jumps in the evaporation zone and in the condensation zone, the thermal resistance of the liquid film above the porous wick in the condensation zone, and, finally, the transport process parameters of the porous wick.

All those parameters, except the last ones, characterize the process of heat and mass transfer inside the heat pipe with the temperature gradient acting as the motive force. The motive force effecting the transfer of liquid through the capillary-porous material is the capillary potential gradient, which appears during evaporation and condensation in various parts of a porous body. The boundary conditions in the condensation zone in low-temperature heat pipes can be of the third, the second, or the first kind.

The maximum thermal power Q_{max} which a heat pipe can transmit is limited, on the one hand, by the critical boiling mode in the wick pores and, on the other hand, by the ability of the porous wick to transmit the liquid when a capillary potential gradient appears. Heat pipes can operate either in the evaporation mode, with the liquid evaporating from the surface of the porous wick, or in the boiling mode. The formula for Q_{max} under conditions of weightlessness during heat removal by evaporation of the liquid from the surface of a porous wick is

$$Q_{\max} = \frac{2\sigma}{R_{\min}} K \frac{\rho_L r'}{\mu_L} \cdot \frac{s}{\left(\frac{L_e}{2} + L_a + \frac{L_c}{2}\right)},$$

based on the equality

$$\Delta p_c = \Delta p_L + \Delta p_V,$$

with the assumption that $\Delta P_V \approx 0$.

The value of Q_{max} at the start of critical boiling is usually found by experiment.

In addition to Q_{max}, one must also know the thermal resistance of a low-temperature heat pipe as well as the temperature drop from the outside surface of the evaporator to the outside surface of the condenser at some fixed value of Q.

We will consider the process of heat and mass transfer in a heat pipe (Fig. 1) whose porous wick is a plate of thickness δ, width b, and length L_T. We assume that heat is removed from the evaporator by the evaporation of liquid at the wick surface. Let us set up the heat balance in an element of porous wick on the evaporator side: thickness dy, width b, and length L_e (Fig. 2).

A current of liquid is driven by the capillary potential gradient into the said element of porous wick, equal to $j_L(y/\delta)$ in the y-direction and equal to $j_L(dy/\delta)$ in the x-direction. Power Q in the y-direction is supplied to the wick element by heat conduction through the wet porous material. This thermal power

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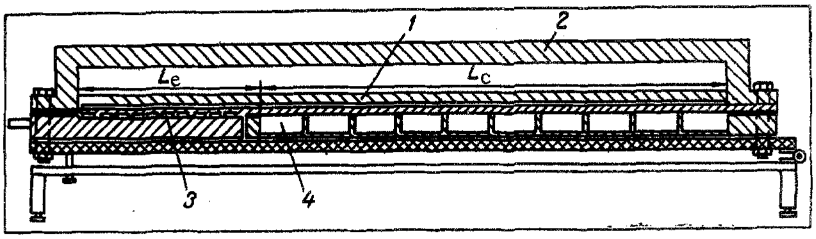

Fig. 1. Longitudinal section through a heat pipe: 1) porous wick in the shape of a plate, length $L_T = L_e + L_c$, width b, thickness δ, 2) top cover, 3) heater, 4) heat exchange ducts on the condenser side.

heats up the entering liquid and the porous wick. Heat is removed by the phase transformation during evaporation of the liquid at the wick surface.

The equation of heat balance for the element of porous wick in the evaporation zone can be set up as follows:

In	Out
$Q_1 + Q_3 = j_L h_L \frac{y}{\delta} + j_L h_L \frac{dy}{\delta}$;	$Q_2' + Q_3' + Q_1'$ $= Q_1 + Q_2 + Q_3 + dQ_1$ $+ dQ_2 + dQ_3$;
$Q_2 = -bL_e\lambda_{eff}\frac{dT}{dy}$;	
$Q_1 + Q_2 + Q_3 = Q_2' + Q_3' + Q_1'$;	$dQ_1 + dQ_2 + dQ_3 = 0.$

(1)

On the basis of the energy balance in this element of porous wick, we write

$$-bL_e\lambda_{eff}\frac{d}{dy}\left(\frac{dT}{dy}\right)dy + j_L\frac{dh_L}{dy}\cdot\frac{ydy}{\delta} = 0. \tag{2}$$

$$-bL_e\lambda_{eff}\frac{d^2T}{dy^2}dy + j_L\frac{y}{\delta}\frac{dh_L}{dy}dy = 0, \tag{3}$$

$$\frac{d^2T}{dy^2} = \frac{j_L}{bL_e\lambda_{eff}\delta}y\frac{dh_L}{dy}. \tag{4}$$

We will now make the following substitution:

$$\frac{dh_L}{dy} = C_{p_L}\frac{dT}{dy} + \frac{1}{\rho_L}\cdot\frac{dp_L}{dy}, \tag{5}$$

where $C_{p.L}$ denotes the specific heat of the porous material filled with liquid.

For wicks in heat pipes dp_L/dy is usually a very small quantity which may be disregarded, so that

$$\frac{dh_L}{dy} \approx C_{p_L}\frac{dT}{dy}. \tag{6}$$

Inserting (6) into (3) yields

$$\frac{d^2T}{dy^2} = \frac{j_L C_{p_L}}{bL_e\delta\lambda_{eff}}y\frac{dT}{dy}. \tag{7}$$

Let $dT/dy = Z$, then

$$\frac{dZ}{dy} = \frac{j_L C_{p_L}}{bL_e\delta\lambda_{eff}}yZ, \tag{8}$$

$$\frac{dZ}{Z} = \frac{j_L C_{p_L}}{bL_e\delta\lambda_{eff}}ydy, \tag{9}$$

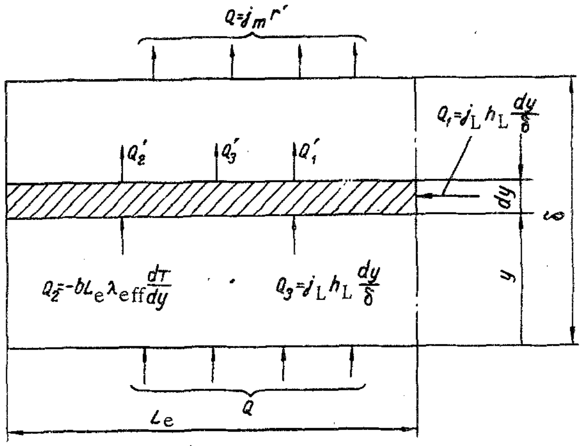

Fig. 2. Heat balance in an element of porous wick, thickness dy, width b, and length L_e on the evaporator side of a heat pipe.

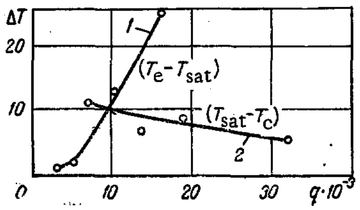

Fig. 3. Temperature drop ΔT(°K) across the porous wick in the evaporator and in the condenser of a heat pipe, as a function of the thermal flux per unit wick surface area q (W/m^2): dots represent test data, curves represent calculated values.

$$\ln C_1 Z = \frac{j_L C_{p_L}}{2bL_e\delta\cdot\lambda_{eff}} y^2, \tag{10}$$

$$Z = \frac{1}{C_1} \exp\left(\frac{j_L C_{p_L}}{2bL_e\delta\lambda_{eff}} y^2\right). \tag{11}$$

The boundary conditions will be stipulated as follows. At y = 0

$$-bL_e\lambda_{eff}\frac{dT}{dy} = Q, \tag{12}$$

$$\frac{1}{C_1} = -\frac{Q}{bL_e\lambda_{eff}}; \quad C_1 = -\frac{bL_e\lambda_{eff}}{Q}, \tag{13}$$

$$\frac{dT}{dy} = -\frac{Q}{bL_e\lambda_{eff}} \exp\left(\frac{j_L C_{p_L}}{2bL_e\delta\cdot\lambda_{eff}} y^2\right). \tag{14}$$

At y = δ we have

$$T = T_{sat}. \tag{15}$$

Then

$$\int_{T_1}^{T_{sat}} dT = -\frac{Q}{bL_e\lambda_{eff}} \int_{y_1}^{\delta} \exp\left(\frac{j_L C_{p_L}}{2bL_e\delta\lambda_{eff}} y^2\right) dy. \tag{16}$$

Integrating the differential equation (4) twice yields the temperature field of a porous wick filled with heat carrier, provided that evaporation occurs at the wick surface.

After integrating twice, we obtain

$$T_1 - T_{sat} = -\frac{Q}{bL_e\lambda_{eff}} \int_{y_1}^{\delta} \exp\left(\frac{j_L C_{p_L}}{2\delta bL_e\lambda_{eff}} y^2\right) dy. \tag{17}$$

The temperature drop across the wick from its outside surface to its inside surface, in the evaporation zone of a heat pipe, is

$$T_e - T_{sat} = -\frac{Q\delta}{bL_e\lambda_{eff}} \int_0^{\delta} \exp\left(\frac{j_L C_{p_L} y^2}{2bL_e\lambda_{eff}^{u}\delta}\right) dy. \tag{18}$$

In an analogous manner we determine the temperature drop across the condenser from the inside surface to the outside surface:

$$T_{sat} - T_c = \frac{Q\delta}{bL_c\lambda_{eff}^{c}} \int_0^{\delta} \exp\left(\frac{-j_L y^2 C_{p_L}}{2bL_c\lambda_{eff}^{c}\delta}\right) dy. \tag{19}$$

The temperature drop across the evaporator sheath and across the condenser sheath is

$$\Delta T_e = \frac{Q}{bL_e\lambda_{sh}}\delta_1; \quad \Delta T_c = \frac{Q}{bL_c\lambda_{sh}}\delta_1. \tag{20}$$

respectively.

The total temperature drop from the outside surface of the evaporator to the outside surface of the condenser in a flat low-temperature heat pipe or steam chamber is

$$T_{out}^{c} - T_{out}^{c} = \Delta T_e + (T_e - T_{sat}) + (T_{sat} - T_c) + \Delta T_c$$
$$= \frac{Q\delta_1}{bL_e\lambda_{sh}} + \frac{Q\delta}{bL_e\lambda_{eff}^{e}} \int_0^{\delta} \exp\left(\frac{j_L y^2 C_{p_L}^{e}}{2bL_e\lambda_{eff}^{e}\delta}\right) dy$$
$$+ \frac{Q\delta}{bL_c\lambda_{eff}^{c}} \int_0^{\delta} \exp\left(\frac{-j_L y^2 C_{p_L}^{c}}{2bL_c\lambda_{eff}^{c}\delta}\right) dy + \frac{Q\delta_1}{bL_c\lambda_{sh}}; \tag{21}$$

$$\int_0^\delta \exp(Ay^2)\,dy = \int_0^\delta \exp\left(\frac{j_L y^2 C_{pL}}{2bL\lambda_{eff}\delta}\right) dy,$$

$$A = \frac{j_L C_{pL}}{2bL\lambda_{eff}\delta}.$$

1. At y < 1

$$\int_0^\delta \exp(Ay^2)\,dy = \delta + \frac{A\delta^3}{3} + \frac{A^2\delta^5}{2!\,5} + \frac{A^3\delta^7}{3!\,7}.$$

A similar analysis for a cylindrical heat pipe [3] yields

$$T_e - T_{sat} = \frac{Q}{2\pi r_{out}^{w} L_e \delta \lambda_{eff}^{e}} \int_0^\delta \exp\left(\frac{j_L C_{pL}^{e}}{4\pi r_{out}^{w} L_e \delta \lambda_{eff}^{e}} y^2\right) dy; \quad (22)$$

$$T_{sat} - T_c = \frac{Q}{2\pi r_{out}^{w} L_c \delta \lambda_{eff}^{c}} \int_0^\delta \exp\left(-\frac{j_L C_{pL}^{c}}{4\pi r_{out}^{w} L_c \delta \lambda_{eff}^{c}} y^2\right) dy. \quad (23)$$

When the boundary conditions at the outside surfaces along the evaporator and the condenser are of the first kind (T_{out}^{e} = const, T_{out}^{c} = const), then the thermal power Q transmitted by the heat pipe can be calculated according to the formula

$$Q = (T_{out}^{e} - T_{out}^{c})\, b \left[\frac{\delta_1}{L_e \lambda_{sh}} + \frac{1}{L_e \lambda_{eff}^{e}} \int_0^\delta \exp\left(\frac{j_L y^2 C_{pL}^{e}}{2bL_e \lambda_{eff}^{e}\delta}\right) dy \right.$$
$$\left. + \frac{1}{L_c \lambda_{eff}^{c}} \int_0^y \exp\left(\frac{-j_L y^2 C_{pL}^{c}}{2bL_c \lambda_{eff}^{c}\delta}\right) dy + \frac{\delta_1}{L_c \lambda_{sh}}\right]^{-1} \leqslant Q_{max}. \quad (24)$$

For verification of these results, the temperature field of a heat pipe with Freon-11, Freon-22, and ethyl alcohol as the heat carrier was measured in special tests [1]. Heat pipe No. 1 was 1.8 m long, had an inside diameter of 19.5 mm, and carried Freon. It had been made of stainless steel. The tube wall was 0.25 mm thick. The porous wick inside was made of glass cloth 3.5 mm thick, with a permeability $K = 0.25 \cdot 10^{11}\,m^2$ and a minimum radius of the interphase surface $R_{min} = 4 \cdot 10^{-5}$ m. The condenser was 250 mm long, the evaporator was 100 mm long.

The test data are compared with the calculations in Fig. 3.

The equivalent thermal conductivity λ_{eff} of porous wicks filled with liquid was calculated according to the formula in [2]:

$$\frac{\lambda_{eff}}{\lambda_{gc}} = \frac{1}{\dfrac{1}{(h/L)^2} + A} + \nu_2(1 - h/L)^2 + \frac{2}{1 + h/l + \dfrac{1}{\nu^2 h/L}}, \quad (25)$$

where

$$A = \frac{1}{\dfrac{\lambda_c}{\lambda_{gc}} + \dfrac{\pi\lambda_{g.z}}{16K_c K_m}\left(\dfrac{h}{L}\right)^2 \cdot 10^3}.$$

NOTATION

σ	is the coefficient of surface tension, kg/sec^2;
R_{min}	is the minimum radius of the liquid–vapor interphase boundary, m;
K	is the permeability of the porous wick, m^2;
ρ_L	is the density of the liquid, kg/m^3;
r'	is the latent heat of evaporation, W · sec/kg;
μ_L	is the dynamic viscosity of the liquid, kg/m · sec;
s = bc	is the cross section area of the wick in a heat pipe, m^2;
b	is the wick width, m;

Π	is the porosity;
L_e	is the evaporator length, m;
L_a	is the length of the adiabatic zone, m;
L_c	is the condenser length, m;
j_L	is the current of liquid through the porous material, kg/sec;
h_L	is the enthalpy of the liquid, J/kg;
λ_{eff}^e, λ_{eff}^c	are the effective thermal conductivity of the porous wick filled with liquid in the evaporation zone and in the condensation zone respectively, W/m·°C;
λ_L^e, λ_L^c	are the thermal conductivity of liquid in the evaporation zone and in the condensation zone respectively, W/m·°C;
λ_{sh}	is the thermal conductivity of the sheath of a heat pipe, W/m·°C;
λ_{gc}	is the thermal conductivity of the wick material (glass cloth), W/m·°C;
$C_{p,L}^e$, $C_{p,L}^c$	are the specific heat of the porous wick filled with liquid in the evaporation zone and in the condensation zone respectively, J/kg·°C;
T_e, T_c	are the temperatures of the porous wick at the surface of contact with the sheath of a heat pipe in the heat supply zone and in the heat removal zone, °C;
T_{sat}	is the saturation temperature in the evaporation zone and in the condensation zone, °C;
T_{out}^e, T_{out}^c	are the temperatures at the surface of a heat pipe in the heat supply zone and in the heat removal zone respectively, °C;
δ	is the thickness of the porous wick, m;
δ_1	is the thickness of the pipe sheath, m;
δ_2	is the thickness of the liquid film on the surface of the porous wick in the condensation zone, m;
r_{out}^w	is the outside radius of the wick in a cylindrical heat pipe, m;
$\delta = r_{out}^w - r_{in}^w$	is the thickness of the wick in a cylindrical heat pipe, m;
v^*	is the dimensionless feed velocity of liquid into the porous wick;
$u_L(x)$	is the feed velocity of liquid into the porous wick, m/sec.

LITERATURE CITED

1. L. L. Vasil'ev, L. P. Grakvich, and S. V. Konev, Inzh. Fiz. Zh., 22, No. 5 (1972).
2. L. L. Vasil'ev and Yu. E. Fraiman, Thermophysical Properties of Poor Heat Conductors [in Russian], Minsk (1967).
3. S. W. Chi and T. A. Cygnarowicz, "Theoretical analyses of cryogenic heat pipes," ASME Paper N HT/SpT-6 (1970).